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DELAY-DEPENDENT STABILITY ANALYSIS AND ROBUST H_∞ CONTROL FOR UNCERTAIN FUZZY SYSTEMS WITH TIME-DELAY

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Abstract

This paper is devoted to the delay-dependent stability analysis and robust H_∞ control for uncertain $T-S$ fuzzy systems with time-delay. New stability conditions are developed for the systems based on the Lyapunov functional approach. Then a design method of the state feedback H_∞ controller is proposed. All the researching results are presented in terms of LMIs. Two numerical examples are given to demonstrate the effectiveness of our proposed methods.

1. Introduction

Since time delays and perturbations are always the sources of instability for a system, the stabilization problems and robust control of nonlinear uncertain systems with time-delay have received considerable attention for decades ([4, 5, 6, 7, 13, 14, 17, 19, 20]). These kind of systems

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can be found in many real life systems, such as electric power systems, large electric networks, rolling mill systems, economic systems, aerospace systems, different types of societal systems and ecological systems. In practice, the inevitable uncertainties may enter a nonlinear system in a much more complex way. The uncertainty may include modeling error, parameter perturbations, fuzzy approximation errors, and external disturbances. So we consider about the uncertain fuzzy systems with time-delay in this paper.

Fuzzy system model and theory [10, 12] have attracted great deal of interests for system analysis and synthesis. It is a useful method to represent complex nonlinear systems by some fuzzy sets and reasoning. When the nonlinear plant is represented by a so-called $T-S$ type fuzzy model, local dynamics in different state-space regions are represented by linear model. The overall model of the system is achieved by fuzzy "blending" of these fuzzy models. Therefore, it has a convenient dynamic structure so that some well-established linear systems theory can be easily applied for theoretical analysis and design of the overall closed-loop controlled system. The control design is carried out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme [11, 16].

Stabilization results for time-delay systems can be classified into two types considering their dependence from time delay. Delay-independent stability condition is independent of the size of the delay. It can be used to study the systems without any information on the time-delays [14, 20]. On the other hand, the delay-dependent stabilization is concerned with the size of the time delay and usually provides an upper bound of the time delay such that the closed-loop system is stable for any time delay less than the upper bound [4, 6, 8, 9]. It is well known that delay-independent criteria often cause conservativeness because of ignoring the information of the size of the delay, especially when the delay is comparatively small. Therefore, in generally speaking, delay-dependent results are less conservative than those for the delay-independent case as the size of the delay is taken into account.

In this paper, we will study the stability and stabilization conditions of uncertain $T-S$ fuzzy systems with time delay. Then we will consider the robust H_∞ controller design method. The major contributions of our paper are summarized as follows. First, it gives a delay-dependent stability condition for uncertain $T-S$ fuzzy systems with time delay by Lyapunov functional approach. Second, based on this result, it proposes a new design method of state feedback robust H_∞ controller. Finally, its results are presented in terms of LMIs.

The paper is organized as follows. In Section 2, the $T-S$ fuzzy model is presented to model an uncertain system with time-delay. In Section 3, by using Lyapunov functional approach, a new delay-dependent stability condition is given in terms of LMIs. In Section 4, the existence condition of a delay-dependent robust H_∞ controller via state feedback is derived. In Section 5, two numerical examples are given to show the effectiveness of our results. The conclusion is drawn in Section 6.

Notation. For a symmetric matrix *X*, the notation $X > 0$ means that the matrix X is positive definite. I is an identity matrix of appropriate dimension. X^T denotes the transpose of matrix *X*. For any nonsingular matrix X, X^{-1} denotes the inverse of matrix *X*. R^n denotes the n-dimensional Euclidean space. $R^{m\times n}$ is the set of all $m\times n$ matrices. $L_2[0, \infty)$ refers to the space of square summable infinite vector sequences. $\|\cdot\|_2$ stands for the usual $L_2[0, \infty)$ norm. $*$ denotes the transposed element in the symmetric position of a matrix.

2. System and Problem Description

Takagi and Sugeno proposed an effective way to represent a fuzzy model of a nonlinear dynamical system. In this paper, we consider a nonlinear time-delay system with parameter uncertainty, which could be described by the following $T-S$ fuzzy time-delay model with n plant rules.

Plant Rule i.

If $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2}, ..., $z_g(t)$ is M_{ig}, then

$$\begin{cases} \dot{x}(t) = (A_{i1} + \Delta A_{i1}(t))x(t) + (A_{i2} + \Delta A_{i2}(t))x(t-\tau) \\ \qquad\quad + (B_i + \Delta B_i(t))u(t) + B_{\omega i}\omega(t), \\ \tilde{z}(t) = C_{i1}x(t) + C_{i2}x(t-\tau) + D_i u(t) \\ x(t) = \varphi(t),\ t \in [-\tau, 0], \end{cases} \tag{1}$$

where $z_1(t), \ldots, z_g(t)$ are the premise variables and $M_{ij}(i = 1, 2, \ldots, n, j = 1, 2, \ldots, g)$ is the fuzzy set; $x(t) \in R^q$ is the state vector; $u(t) \in R^m$ is the input vector; $\omega(t)$ is the disturbance which belongs to $L_2[0, \infty)$; $\tilde{z}(t) \in R^p$ is the controlled output; $\tau > 0$ is a real positive constant representing the time-delay of the fuzzy system; $\varphi(t)$ is the initial condition of system (1); $A_{i1}, A_{i2}, B_i, B_{\omega i}, C_{i1}, C_{i2}$ and $D_i (i = 1, 2, \ldots, n)$ are constant matrices of appropriate dimensions; $\Delta A_{i1}(t), \Delta A_{i2}(t)$ and $\Delta B_{i1}(t) (i = 1, 2, \ldots, n)$ are realvalued unknown matrices representing time-varying parameter uncertainties of (1) and they satisfy the following assumption.

Assumption 1.

$$[\Delta A_{i1}(t), \Delta A_{i2}(t), \Delta B_i(t)] = U_i F_i(t) [E_{i1}, E_{i2}, E_i], \tag{2}$$

where E_{i1}, E_{i2}, E_i and $U_i (i = 1, 2, \ldots, n)$ *are known real constant matrices of appropriate dimensions*; $F_i(t) (i = 1, 2, \ldots, n)$ *are unknown real time-varying matrices with Lebesgue measurable elements satisfying*

$$F_i(t)^T F_i(t) \leq I, \qquad i = 1, 2, \ldots, n. \tag{3}$$

Let $\mu_i(z(t))$ be the normalized membership function of the inferred fuzzy set $\rho_i(z(t))$, i.e.,

$$\mu_i(z(t)) = \frac{\rho_i(z(t))}{\sum_{i=1}^{n} \rho_i(z(t))},$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_g(t)]$, $\rho_i(z(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t))$, $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Then, it can be seen that

$$\rho_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, n, \sum_{i=1}^{n} \rho_i(z(t)) > 0, \quad \forall t \geq 0.$$

Therefor, for all $t \geq 0$,

$$\mu_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, n, \sum_{i=1}^{n} \mu_i(z(t)) = 1. \tag{4}$$

By using the center-average defuzzifier, product inference and singleton fuzzifier, the $T - S$ fuzzy model (1) can be expressed by the following global model.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t))[\widetilde{A}_{i1}x(t) + \widetilde{A}_{i2}x(t-\tau) + \widetilde{B}_i u(t) + B_{\omega i}\omega(t)], \\ \widetilde{z}(t) = \sum_{i=1}^{n} \mu_i(z(t))[C_{i1}x(t) + C_{i2}x(t-\tau) + D_i u(t)], \\ x(t) = \varphi(t), \ t \in [-\tau, 0]. \end{cases} \tag{5}$$

where $\widetilde{A}_{i1} \triangleq A_{i1} + \Delta A_{i1}(t)$, $\widetilde{A}_{i2} \triangleq A_{i2} + \Delta A_{i2}(t)$, $\widetilde{B}_i \triangleq B_i + \Delta B_i(t)$, $i = 1, 2, \ldots, n$.

By the parallel distributed compensation (PDC) technique, we consider the following $T - S$ fuzzy-model-based state feedback controller for the fuzzy system (5). The ith controller rule is

If $z_1(t)$ is M_{i1}, $z_2(t)$ is M_{i2}, ..., $z_g(t)$ is M_{ig}, then

$$u(t) = K_i x(t), \tag{6}$$

where $K_i (i = 1, 2, \ldots, n)$ is the controller gain of (6) to be determined. Then, the overall fuzzy state feedback controller is given by

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t)) K_i x(t). \tag{7}$$

By (5) and (7), we can obtain the following closed-loop fuzzy system

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i(z(t))\mu_j(z(t))[(\widetilde{A}_{i1} + \widetilde{B}_i K_j)x(t) + \widetilde{A}_{i2}x(t-\tau) + B_{\omega i}\omega(t)], \\ \widetilde{z}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i(z(t))\mu_j(z(t))[(C_{i1} + D_i K_j)x(t) + C_{i2}x(t-\tau)], \\ x(t) = \varphi(t),\ t \in [-\tau, 0]. \end{cases} \tag{8}$$

3. Delay-dependent Stability Analysis

In this section, we consider the problem of delay-dependent stabilization of the following closed-loop fuzzy system. For simplicity, let $\mu_i = \mu_i(z(t))$,

$$\dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j[(\widetilde{A}_{i1} + \widetilde{B}_i K_j)x(t) + \widetilde{A}_{i2}x(t-\tau)]. \tag{9}$$

Using the Newton-Leibniz formula, we have

$$\int_{-\tau}^{0}\dot{x}(t+\theta)d\theta = x(t) - x(t-\tau).$$

Then, an equivalent form of fuzzy system (9) is that

$$\dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\left[(\widetilde{A}_{i1} + \widetilde{A}_{i2} + \widetilde{B}_i K_j)x(t) - \widetilde{A}_{i2}\int_{-\tau}^{0}\dot{x}(t+\theta)d\theta\right]. \tag{10}$$

It is easy to see that systems (9) and (10) have a common solution. Thus, the stability problem of (9) can be transformed to the same problem of (10).

Three important lemmas should be presented because they are the key to prove the theorems.

Lemma 1 [15]**.** *For any two matrices X and Y, we have*

$$X^T Y + Y^T X \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y,$$

where $X \in R^{m\times n}$, $Y \in R^{m\times n}$, and ε is any positive constant.

Lemma 2 [1]. *For any two vectors* $x(t),\ y(t) \in R^n$, *we have*

$$2x^T(t)y(t) \leq x^T(t)G^{-1}x(t) + y^T(t)Gy(t),$$

where $G \in R^{n \times n}$ *and* $G > 0$.

Lemma 3 [18]. *Given the matrices Y, U, and E of appropriate dimensions where* $Y = Y^T$, *then for any matrix F satisfying* $F^T F \leq I$,

$$Y + UFE + E^T F^T U^T < 0$$

holds if and only if there exists a constant $\varepsilon > 0$ *satisfying*

$$Y + \varepsilon UU^T + \varepsilon^{-1} E^T E < 0.$$

Based on the Lyapunov functional approach, the delay-dependent stabilization result of $T - S$ fuzzy system (9) is summarized in the following theorem.

Theorem 1. *For* $\tau > 0$, *if there exist matrices* $X > 0,\ M > 0,\ N > 0$ *and* Y_i *satisfying the following LMIs*:

$$\begin{bmatrix} \widetilde{\Phi}_{ii} & * & * & * & * \\ 0 & -M & * & * & * \\ \widetilde{A}_{i1}X + \widetilde{B}_i Y_i & \widetilde{A}_{i2}M & -\tau^{-1}N & * & * \\ N\widetilde{A}_{i2}^T & 0 & 0 & -\tau^{-1}N & * \\ X & 0 & 0 & 0 & -M \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} 2(\widetilde{\Phi}_{ij} + \widetilde{\Phi}_{ji}) & * & * & * & * \\ 0 & -4M & * & * & * \\ (\widetilde{A}_{i1} + \widetilde{A}_{j1})X + \widetilde{B}_i Y_j + \widetilde{B}_j Y_i & (\widetilde{A}_{i2} + \widetilde{A}_{j2})M & -\tau^{-1}N & * & * \\ N(\widetilde{A}_{i2} + \widetilde{A}_{j2})^T & 0 & 0 & -\tau^{-1}N & * \\ X & 0 & 0 & 0 & -0.25M \end{bmatrix} < 0, \tag{12}$$

then the closed-loop fuzzy system (9) *is asymptotically stable, where* $i = 1, 2, \ldots, n$ *in* (11) *and* $1 \leq i < j \leq n$ *in* (12), $\widetilde{\Phi}_{ij} = (\widetilde{A}_{i1} + A_{i2})X + X(\widetilde{A}_{i1} + \widetilde{A}_{i2})^T + \widetilde{B}_i Y_j + Y_j^T \widetilde{B}_i^T$. *Moreover, the control gain* K_i *is given by* $K_i = Y_i X^{-1}$, $i = 1, 2, \ldots, n$.

Proof. Let $W_{ij} = \widetilde{A}_{i1} + \widetilde{A}_{i2} + \widetilde{B}_i K_j$ and $R_{ij} = \widetilde{A}_{i1} + \widetilde{B}_i K_j$. Choose Lyapunov function as follows:

$$V(x(t)) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S\dot{x}(s)dsd\theta, \quad (13)$$

where $P > 0$, $Q > 0$ and $S > 0$. Then, the derivative of (13) along the trajectory of the closed-loop system (9) (or (10)) is given by

$$\dot{V}(x(t)) = 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) + \tau\dot{x}^T(t)S\dot{x}(t) - \int_{-\tau}^{0} \dot{x}^T(t+\theta)S\dot{x}(t+\theta)d\theta$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\mu_j\Big[2x^T(t)PW_{ij}x(t) - 2x^T(t)P\widetilde{A}_{i2}\int_{-\tau}^{0}\dot{x}(t+\theta)d\theta + x^T(t)Qx(t)$$

$$- x^T(t-\tau)Qx(t-\tau) + \tau\dot{x}^T(t)S\dot{x}(t) - \int_{-\tau}^{0}\dot{x}^T(t+\theta)S\dot{x}(t+\theta)d\theta\Big]. \quad (14)$$

By (9), we have

$$\tau\dot{x}^T(t)S\dot{x}(t) = \tau\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \mu_i\mu_j\mu_k\mu_l[R_{ij}x(t) + \widetilde{A}_{i2}x(t-\tau)]^T S[R_{kl}x(t) + \widetilde{A}_{k2}x(t-\tau)]$$

$$\leq \tau\sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\mu_j[R_{ij}x(t) + \widetilde{A}_{i2}x(t-\tau)]^T S[R_{ij}x(t) + \widetilde{A}_{i2}x(t-\tau)]$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\mu_j[x^T(t) \;\; x^T(t-\tau)]\begin{bmatrix} R_{ij}^T \\ \widetilde{A}_{i2}^T \end{bmatrix}(\tau S)[R_{ij} \;\; \widetilde{A}_{i2}]\begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}. \quad (15)$$

By Lemma 2, we have

$$-\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j 2x^T(t)P\widetilde{A}_{i2}\int_{-\tau}^{0}\dot{x}(t+\theta)d\theta$$

$$= -\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\int_{-\tau}^{0}2x^T(t)P\widetilde{A}_{i2}\dot{x}(t+\theta)d\theta$$

$$\leq \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\mu_j\left[\tau x^T(t)P\widetilde{A}_{i2}S^{-1}\widetilde{A}_{i2}^TPx(t) + \int_{-\tau}^{0}\dot{x}^T(t+\theta)S\dot{x}(t+\theta)d\theta\right]. \quad (16)$$

By substituting inequalities (15) and (16) into (14), we can obtain

$$\dot{V}(x(t)) \leq \sum_{i=1}^{n}\mu_1^2[x^T(t) \quad x^T(t-\tau)]\Omega_{ii}\begin{bmatrix} x(t) \\ x(t-\tau)\end{bmatrix}$$

$$+ 0.5\sum_{i=1}^{n-1}\sum_{j>1}^{n}\mu_i\mu_j[x^T(t) \quad x^T(t-\tau)]\hat{\Omega}_{ij}\begin{bmatrix} x(t) \\ x(t-\tau)\end{bmatrix},$$

where

$$\Omega_{ii} = \begin{bmatrix} PW_{ii} + W_{ii}^TP + Q + \tau R_{ii}^TSR_{ii} + \tau P\widetilde{A}_{i2}S^{-1}\widetilde{A}_{i2}^TP & * \\ \tau\widetilde{A}_{i2}^TSR_{ii} & -Q + \tau\widetilde{A}_{i2}^TS\widetilde{A}_{i2}\end{bmatrix},$$

$$\hat{\Omega}_{ij} = \begin{bmatrix} \hat{\Omega}_{11}^{ij} & * \\ \tau(\widetilde{A}_{i2} + \widetilde{A}_{j2})^TS(R_{ij} + R_{ji}) & -4Q + \tau(\widetilde{A}_{i2} + \widetilde{A}_{j2})^TS(\widetilde{A}_{i2} + \widetilde{A}_{j2})\end{bmatrix},$$

$$\hat{\Omega}_{11}^{ij} = 2P(W_{ij} + W_{ji}) + 2(W_{ij} + W_{ji})^TP + 4Q + \tau(R_{ij} + R_{ji})^TS(R_{ij} + R_{ji})$$

$$+ \tau P(\widetilde{A}_{i2} + \widetilde{A}_{j2})S^{-1}(\widetilde{A}_{i2} + \widetilde{A}_{j2})^TP.$$

By using Schur complement, we can get the following conditions for $\dot{V}(x(t)) < 0$.

$$\begin{bmatrix} PW_{ii} + W_{ii}^TP & * & * & * & * \\ 0 & -Q & * & * & * \\ R_{ii} & \widetilde{A}_{i2} & -\tau^{-1}S^{-1} & * & * \\ \widetilde{A}_{i2}^TP & 0 & 0 & -\tau^{-1}S & * \\ I & 0 & 0 & 0 & -Q^{-1}\end{bmatrix} < 0, \quad (17)$$

and

$$\begin{bmatrix} 2P(W_{ij}+W_{ji})+2(W_{ij}+W_{ji})^T P & * & * & * & * \\ 0 & -Q & * & * & * \\ R_{ij}+R_{ji} & \tilde{A}_{i2}+\tilde{A}_{j2} & -\tau^{-1}S^{-1} & * & * \\ (\tilde{A}_{i2}+\tilde{A}_{j2})^T P & 0 & 0 & -\tau^{-1}S & * \\ I & 0 & 0 & 0 & -0.25Q^{-1} \end{bmatrix} < 0. \tag{18}$$

Define $X = P^{-1}$, $M = Q^{-1}$, $N = S^{-1}$ and $Y_i = K_i X$. Then pre and post-multiplying (17) and (18) by $diag\{X \quad M \quad I \quad N \quad I\}$, we can obtain (11) and (12). This completes the proof.

Note that the matrices in (11) and (12) are monotonic increasing with respect to $\tau > 0$. Therefore, we can get the following theorem.

Theorem 2. *For a given scalar* τ_M *such that* $\tau \in [0, \tau_M]$, *if there exist matrices* $X > 0$, $M > 0$, $N > 0$ *and* Y_i *satisfying the following LMIs*:

$$\begin{bmatrix} \tilde{\Phi}_{ii} & * & * & * & * \\ 0 & -M & * & * & * \\ \tilde{A}_{i1}X+\tilde{B}_i Y_i & \tilde{A}_{i2}M & -\tau_M^{-1}N & * & * \\ N\tilde{A}_{i2}^T & 0 & 0 & -\tau_M^{-1}N & * \\ X & 0 & 0 & 0 & -M \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} 2(\tilde{\Phi}_{ij}+\tilde{\Phi}_{ji}) & * & * & * & * \\ 0 & -4M & * & * & * \\ (\tilde{A}_{i1}+\tilde{A}_{j1})X+\tilde{B}_i Y_j+\tilde{B}_j Y_i & (\tilde{A}_{i2}+\tilde{A}_{j2})M & -\tau_M^{-1}N & * & * \\ N(\tilde{A}_{i2}+\tilde{A}_{j2})^T & 0 & 0 & -\tau_M^{-1}N & * \\ X & 0 & 0 & 0 & -0.25M \end{bmatrix} < 0, \tag{20}$$

then the closed-loop fuzzy system (9) *is asymptotically stable, where* $i = 1, 2, \ldots, n$ *in* (19) *and* $1 \le i < j \le n$ *in* (20).

Remark 1. *If system* (9) *has no parameter uncertainties*, *i.e.*, $\Delta A_{i1}(t) = \Delta A_{i2}(t) = \Delta B_i(t) = 0,$ *then Theorem* 2 *is the delay-dependent stability criterion of* $T - S$ *fuzzy time-delay systems.*

In the following, we will consider about the parameter uncertainties.

Theorem 3. *For a given scalar* τ_M *such that* $\tau \in [0, \tau_M],$ *if there exist matrices* $X > 0, M > 0, N > 0, Y_i$ *and positive constants* ε_{ij} *satisfying the following LMIs*:

$$\begin{bmatrix} \Theta_{11}^{ii} & * \\ \Theta_{21}^{ii} & \Theta_{22}^{ii} \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} \hat{\Theta}_{11}^{ij} & * & * \\ \hat{\Theta}_{21}^{ij} & \Theta_{22}^{ij} & * \\ \hat{\Theta}_{21}^{ji} & 0 & \Theta_{22}^{ji} \end{bmatrix} < 0, \tag{22}$$

then the closed-loop fuzzy system (9) *is asymptotically stable*, *where* $i = 1, 2, \ldots, n$ *in* (21) *and* $1 \le i < j \le n$ *in* (22),

$$\Theta_{11}^{ii} = \begin{bmatrix} \Phi_{11}^{ii} & * & * & * & * \\ 0 & -M & * & * & * \\ A_{i1}X + B_iY_i & A_{i2}M & -\tau_M^{-1}N + \varepsilon_{ii}U_iU_i^T & * & * \\ NA_{i2}^T & 0 & 0 & -\tau_M^{-1}N & * \\ X & 0 & 0 & 0 & -M \end{bmatrix},$$

$$\Theta_{21}^{ii} = \begin{bmatrix} E_{i1}X + E_{i2}X + E_iY_i & 0 & 0 & E_{i2}N & 0 \\ E_{i1}X + E_iY_i & E_{i2}M & 0 & 0 & 0 \end{bmatrix},$$

$$\Theta_{22}^{ii} = \begin{bmatrix} -\varepsilon_{ij}I & * \\ 0 & -\varepsilon_{ij}I \end{bmatrix},$$

$$\hat{\Theta}_{11}^{ij}=\begin{bmatrix} \hat{\Phi}_{11}^{ij} & * & * & * & * \\ 0 & -4M & * & * & * \\ \hat{\Phi}_{31}^{ij} & (A_{i2}+A_{j2})M-\tau_M^{-1}N+\varepsilon_{ij}U_iU_i^T+\varepsilon_{ji}U_jU_j^T & * & * & * \\ N(A_{i2}+A_{j2})^T & 0 & 0 & -\tau_M^{-1}N & * \\ X & 0 & 0 & 0 & -0.25M \end{bmatrix},$$

$$\hat{\Theta}_{21}^{ij} = \begin{bmatrix} E_{i1}X + E_{i2}X + E_iY_j & 0 & 0 & 0.5E_{i2}N & 0 \\ E_{i1}X + E_iY_j & E_{i2}M & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{11}^{ii} = (A_{i1} + A_{i2})X + X(A_{i1} + A_{i2})^T + B_iY_i + Y_i^TB_i^T + \varepsilon_{ii}U_iU_i^T,$$

$$\hat{\Phi}_{11}^{ij} = 2(A_{i1} + A_{i2} + A_{j1} + A_{j2})X + 2X(A_{i1} + A_{i2} + A_{j1} + A_{j2})^T$$

$$+2(B_iY_j + B_jY_i) + 2(B_iY_j + B_jY_i)^T + 2\varepsilon_{ij}U_iU_i^T + 2\varepsilon_{ji}U_jU_j^T,$$

$$\Phi_{31}^{ii} = (A_{i1} + A_{i1}) + B_iY_j + B_jY_i.$$

Moreover, the state feedback controller gain of (7) *is given by* $K_i = Y_iX^{-1}$, $i = 1, 2, \ldots, n$.

Proof. Replacing $\tilde{A}_{i1}$, $\tilde{A}_{i2}$ and $\tilde{B}_i$ with $A_{i1} + U_iF_i(t)E_{i1}$, $A_{i2} + U_iF_i(t)E_{i2}$ and $B_i + U_iF_i(t)E_i$ in (19) and (20), respectively, by Assumption 1 and Lemma 3, using Schur complements, we can obtain (21) and (22).

4. Design Method of State Feedback H_∞ Controller

Now we consider the state feedback H_∞ controller design of system (5). For H_∞ control, we always consider the following performance index

$$J(\omega) \triangleq \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2\omega^T(\theta)\omega(\theta)]d\theta \tag{23}$$

under zero initial condition, where $\gamma > 0$ is a prescribed constant.

Remark 2. *The purpose to design a delay-dependent robust* H_∞ *controller* (7) *for* $T-S$ *fuzzy system* (5) *such that for all admissible uncertainties satisfying* (2), (3) *and for a prescribed constant* $\gamma > 0$,

[a] *The closed-loop fuzzy system* (8) *is asymptotically stable when* $\omega(t) = 0$;

[b] *The closed-loop fuzzy system* (8) *satisfies* $\| \tilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$, *i.e.* $J(\omega) < 0$ *for all nonzero* $\omega(t) \in L_2[0, \infty)$ *under zero initial condition.*

We consider the performance index (23) in the following theorem.

Theorem 4. *For a prescribe constant* $\gamma > 0$ *and a scalar* $\tau_M > 0$ *such that* $\tau \in [0, \tau_M]$, *if there exist* $X > 0, M > 0, N > 0$ *and* Y_i *satisfying the following LMIs*:

$$\begin{bmatrix} \tilde{\Phi}_{ii} & * & * & * & * & * & * \\ 0 & -M & * & * & * & * & * \\ B_{\omega i}^T & 0 & -\gamma^2 I & * & * & * & * \\ \tilde{A}_{i1}X + \tilde{B}_i Y_i & \tilde{A}_{i2}M & B_{\omega i} & -\tau_M^{-1}N & * & * & * \\ N\tilde{A}_{i2}^T & 0 & 0 & 0 & -\tau_M^{-1}N & * & * \\ C_{i1}X + D_i Y_i & C_{i2}M & 0 & 0 & 0 & -I & * \\ X & 0 & 0 & 0 & 0 & 0 & -M \end{bmatrix} < 0, \quad (24)$$

$$\begin{bmatrix} 2(\tilde{\Phi}_{ij} + \tilde{\Phi}_{ji}) & * & * & * & * & * & * \\ 0 & -4M & * & * & * & * & * \\ 2(B_{\omega i} + B_{\omega j})^T & 0 & -4\gamma^2 I & * & * & * & * \\ (\tilde{A}_{i1} + \tilde{A}_{j1})X + \tilde{B}_i Y_j + \tilde{B}_j Y_i & (\tilde{A}_{i2} + \tilde{A}_{j2})M & B_{\omega i} + B_{\omega j} & -\tau_M^{-1}N & * & * & * \\ N(\tilde{A}_{i2} + \tilde{A}_{j2})^T & 0 & 0 & 0 & -\tau_M^{-1}N & * & * \\ (C_{i1} + C_{j1})X + D_i Y_j + D_j Y_i & (C_{i2} + C_{j2})M & 0 & 0 & 0 & -I & * \\ X & 0 & 0 & 0 & 0 & 0 & -0.25M \end{bmatrix} < 0, \quad (25)$$

then $j(\omega) < 0$, *where* $i = 1, 2, \ldots, n$ *in* (24), $1 \le i < j \le n$ *in* (25), *and* $\tilde{\Phi}_{ij}$ *is given in Theorem* 1.

Proof. Under initial condition, we have

$$J(\omega) = \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2\omega^T(\theta)\omega(\theta) + \dot{V}(x(\theta))]d\theta - V(x(\infty))$$

$$\le \int_0^\infty [\tilde{z}^T(\theta)\tilde{z}(\theta) - \gamma^2\omega^T(\theta)\omega(\theta) + \dot{V}(x(\theta))]d\theta$$

$$= \sum_{i=1}^{n} \mu_i^2 [x^T(t) \quad x^T(t-\tau) \quad \omega(t)]\Gamma_{ii} \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}$$

$$+ 0.5 \sum_{i=1}^{n-1} \sum_{j>i}^{n} \mu_i\mu_j [x^T(t) \quad x^T(t-\tau) \quad \omega(t)]\hat{\Gamma}_{ij} \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix},$$

where

$$\Gamma_{ij} = \begin{bmatrix} \Xi^{ii} & * & * \\ \tau\tilde{A}_{i2}^T S R_{ii} & -Q + \tau\tilde{A}_{i2}^T S\tilde{A}_{i2} + C_{i2}^T C_{i2} & * \\ \tau B_{\omega i}^T S R_{ii} + B_{\omega i}^T P & \tau B_{\omega i}^T S\tilde{A}_{i2} & \tau B_{\omega i}^T S B_{\omega i} - \gamma^2 I \end{bmatrix},$$

$$\hat{\Gamma}_{ij} = \begin{bmatrix} \hat{\Xi}_{11}^{ij} & * & * \\ \hat{\Xi}_{21}^{ij} & \hat{\Xi}_{22}^{ij} & * \\ \hat{\Xi}_{31}^{ij} & \hat{\Xi}_{32}^{ij} & \hat{\Xi}_{33}^{ij} \end{bmatrix},$$

$$\Xi^{ii} = PW_{ii} + W_{ii}^T P + Q + \tau R_{ii}^T S R_{ii} + \tau P\tilde{A}_{i2} S^{-1} \tilde{A}_{i2}^T P$$
$$+ (C_{i1} + D_i K_i)^T (C_{i1} + D_i K_i),$$

$$\hat{\Xi}_{11}^{ij} = 2P(W_{ij} + W_{ji}) + 2(W_{ij} + W_{ji})^T P + 4Q + \tau(R_{ij} + R_{ji})^T S(R_{ij} + R_{ji})$$
$$+ \tau P(\tilde{A}_{i2} + \tilde{A}_{j2})S^{-1}(\tilde{A}_{i2} + \tilde{A}_{j2})^T P$$
$$+ (C_{i1} + C_{j1} + D_i K_j + D_j K_i)^T (C_{i1} + C_{j1} + D_i K_j + D_j K_i),$$

$$\hat{\Xi}_{21}^{ij} = \tau(\tilde{A}_{i2} + \tilde{A}_{j2})^T S(R_{ij} + R_{ji}),$$

$$\hat{\Xi}_{22}^{ij} = -4Q + \tau(\tilde{A}_{i2} + \tilde{A}_{j2})^T S(\tilde{A}_{i2} + \tilde{A}_{j2}) + (C_{i2} + C_{j2})^T (C_{i2} + C_{j2}),$$

$$\hat{\Xi}_{31}^{ij} = \tau(B_{\omega i} + B_{\omega j})^T S(R_{ij} + R_{ji}) + 2(B_{\omega i} + B_{\omega j})^T P,$$

$$\hat{\Xi}_{32}^{ij} = \tau(B_{\omega i} + B_{\omega j})^T S(\widetilde{A}_{i2} + \widetilde{A}_{j2}),$$

$$\hat{\Xi}_{33}^{ij} = \tau(B_{\omega i} + B_{\omega j})^T S(B_{\omega i} + B_{\omega j}) - 4\gamma^2 I.$$

By Theorem 2 and the proof of Theorem 1, we can obtain that when LMIs (24) and (25) hold, $J(\omega) < 0$.

Remark 3. *It is easy to see that* (24) *implies* (19), *and* (25) *implies* (20).

By Remark 2, Remark 3 and Theorem 4, considering about the uncertainties, the design method of delay-dependent robust H_∞ controller is obtained in the following theorem.

Theorem 5. *For a prescribed scalar* $\gamma > 0$ *and a scalar* $\tau_M > 0$ *such that* $\tau \in [0, \tau_M]$, $T - S$ *fuzzy system* (8) *is stable and satisfies* $\| \widetilde{z}(t) \|_2 < \gamma \| \omega(t) \|_2$ *for all nonzero* $\omega(t) \in L_2[0, \infty]$ *under zero initial condition if there exist* $X > 0, M > 0, N > 0, Y_i (i = 1, 2, \ldots, n)$ *of appropriate dimensions and positive constants* ε_{ij} *such that the following LMIs simultaneously hold.*

$$\begin{bmatrix} \Lambda_{11}^{ij} & * \\ \Lambda_{21}^{ii} & \Lambda_{22}^{ii} \end{bmatrix} < 0, \tag{26}$$

$$\begin{bmatrix} \hat{\Lambda}_{11}^{ij} & * & * \\ \hat{\Lambda}_{21}^{ij} & \Lambda_{22}^{ij} & * \\ \hat{\Lambda}_{21}^{ji} & 0 & \Lambda_{22}^{ji} \end{bmatrix} < 0, \tag{27}$$

where $i = 1, 2, \ldots, n$ *in* (26), $1 \le i < j \le n$ *in* (27),

$$\Lambda_{11}^{ii} = \begin{bmatrix} \Psi_{11}^{ii} & * & * & * & * & * & * \\ 0 & -M & * & * & * & * & * \\ B_{\omega 1}^T & 0 & -\gamma^2 I & * & * & * & * \\ A_{i1}X + B_i Y_i & A_{i2}M & B_{\omega i} & -\tau_M^{-1} N + \varepsilon_{ii} U_i U_i^T & * & * & * \\ N A_{i2}^T & 0 & 0 & 0 & -\tau_M^{-1} N & * & * \\ C_{i1}X + D_i Y_i & C_{i2}M & 0 & 0 & 0 & -I & * \\ X & 0 & 0 & 0 & 0 & 0 & -M \end{bmatrix},$$

$$\Lambda_{21}^{ii} = \begin{bmatrix} E_{i1}XE_{i2} + X + E_iY_i & 0 & 0 & 0 & E_{i2}N & 0 & 0 \\ E_iX + E_iY_i & E_{i2}M & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Lambda_{22}^{ij} = \Theta_{22}^{ij},$$

$$\hat{\Lambda}_{11}^{ij} = \begin{bmatrix} \hat{\Psi}_{11}^{ij} & * & * & * & * & * & * \\ 0 & -4M & * & * & * & * & * \\ 2(B_{\omega i}+B_{\omega j})^T & 0 & -4\gamma^2 I & * & * & * & * \\ \hat{\Psi}_{41}^{ij} & (A_{i2}+A_{j2})M & B_{\omega i}+B_{\omega j} & \hat{\Psi}_{44}^{ij} & * & * & * \\ N(A_{i2}+A_{j2})^T & 0 & 0 & 0 & -\tau_M^{-1}N & * & * \\ \hat{\Psi}_{61}^{ij} & (C_{i2}+C_{j2})M & 0 & 0 & 0 & -I & * \\ X & 0 & 0 & 0 & 0 & 0 & -0.25M \end{bmatrix},$$

$$\hat{\Lambda}_{21}^{ij} = \begin{bmatrix} E_{i1}X + E_{i2}X + E_iY_j & 0 & 0 & 0 & 0.5E_{i2}N & 0 & 0 \\ E_{i1}X + E_iY_j & E_{i2}M & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{11}^{ii} = (A_{i1} + A_{i2})X + X(A_{i1} + A_{i2})^T X + B_iY_i + Y_i^T B_i^T + \varepsilon_{ii}U_iU_i^T,$$

$$\hat{\Psi}_{11}^{ij} = 2(A_{i1} + A_{i2} + A_{j1} + A_{j2})X + 2X(A_{i1} + A_{i2} + A_{j1} + A_{j2})^T + 2(B_iY_j + B_jY_i) + 2(B_iY_j + B_jY_i)^T + 2\varepsilon_{ij}U_iU_i^T + 2\varepsilon_{ji}U_jU_j^T,$$

$$\hat{\Psi}_{41}^{ij} = (A_{i1} + A_{j1})X + B_iY_j + B_jY_i,$$

$$\hat{\Psi}_{44}^{ij} = -\tau_M^{-1}N + \varepsilon_{ij}U_iU_i^T + \varepsilon_{ji}U_jU_j^T,$$

$$\hat{\Psi}_{61}^{ij} = (C_{i1} + C_{j1})X + D_iY_j + D_jY_i.$$

Moreover, the state feedback controller gain of (7) *is given by* $K_i = Y_iX^{-1}$, $i = 1, 2, \ldots, n$.

Proof. Replacing $\widetilde{A}_{i1}$, $\widetilde{A}_{i2}$ and $\widetilde{B}_i$ with $A_{i1} + U_iF_i(t)E_{i1}$, $A_{i2} + U_iF_i(t)E_{i2}$ and $B_i + U_iF_i(t)E_i$ in (24) and (25), respectively, by Assumption 1 and Lemma 3, using Schur complements, we can obtain (26) and (27). By Remark 3, we know that when (26) and (27) hold, system (8) with $\omega(t) = 0$ is asymptotically stable. By Remark 2, we can complete the proof.

5. Numerical Examples

In this section, two examples are used to illustrate the proposed methods.

Example 1. Consider an uncertain $T - S$ fuzzy system with time-delay as follows:

Rule 1. If $x_2(t)$ is *small*, then

$$\dot{x}(t) = (A_{11} + \Delta A_{11})x(t) + A_{12}x(t - \tau) + B_1 u(t).$$

Rule 2. If $x_2(t)$ is *big*, then

$$\dot{x}(t) = (A_{21} + \Delta A_{21})x(t) + A_{22}x(t - \tau) + B_2 u(t). \tag{28}$$

The model parameters are given as follows:

$$A_{11} = \begin{bmatrix} 0 & -0.9 \\ -0.3 & -1.5 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0.01 \\ -0.018 & 0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0 & -0.8 \\ -0.4 & -1.7 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & 0.01 \\ -0.012 & 0.19 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad E_{11} = [0.2 \quad 0.2], \quad F_1(t) = \sin(t),$$

$$U_2 = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}, \quad E_{21} = [-0.2 \quad 0.2], \quad F_2(t) = -\sin(t),$$

where $\Delta A_{i1} = U_i F_i(t) E_{i1} (i = 1, 2)$.

The membership function for x_2 are as follows:

$$small(x_2) = \begin{cases} 1, & x_2 \in (-\infty, -1], \\ 0.5(1 - x), & x_2 \in [-1, 1], \\ 0, & x_2 \in [1, \infty), \end{cases}$$

and

$$big(x_2) = \begin{cases} 0, & x_2 \in (-\infty, -1], \\ 0.5(1 + x), & x_2 \in [-1, 1], \\ 1, & x_2 \in [1, \infty). \end{cases}$$

The system (28) with $u(t) = 0$, $\tau = 0.5$ has unstable response as shown in Figure 1 for the initial condition $x(0) = [2 - 5]^T$.

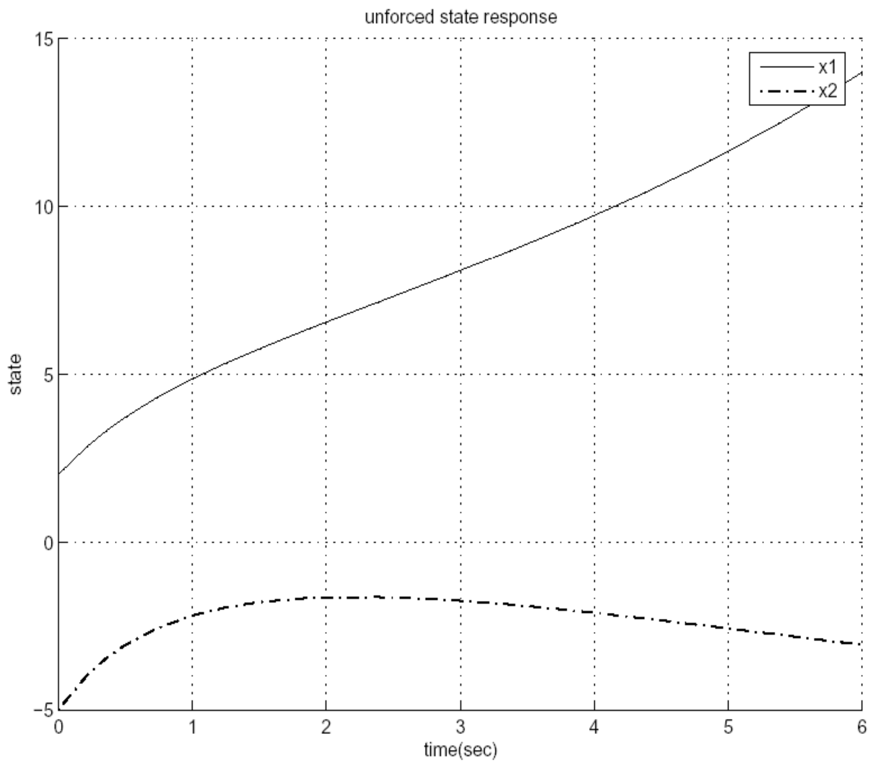

Figure1. The response of the system (28) with $u(t) = 0$ and initial condition $x(0) = [2 - 5]^T$, and $\tau = 0.5$.

Hereby, if we give the state feedback controller as (7), the closed-loop fuzzy system can be obtain. Then solving the LMIs (21) and (22) by MATLAB LMI Toolbox, the state feedback gain $K_i (i = 1, 2)$ can be obtain as follows:

$$K_1 = [-1.2665 \;\; 0.6977] \quad K_2 = [-1.2445 \;\; 0.7281].$$

The simulation result with initial condition $x(0) = [2, -5]^T$ is shown in Fig. 2.

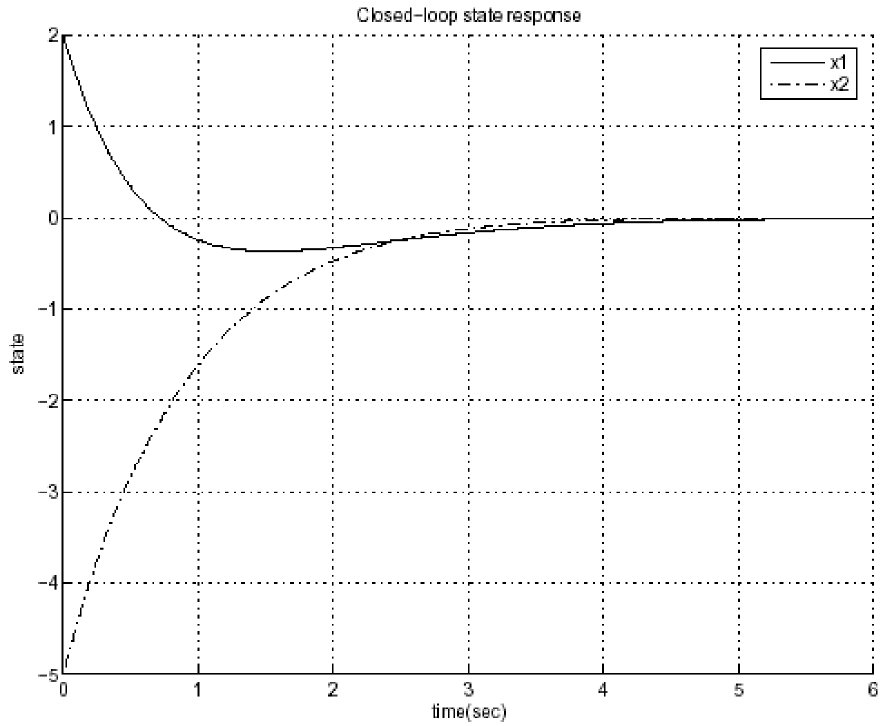

Figure 2. State response under initial condition $x(0) = [2, -5]^T$ and $\tau = 0.5$.

Example 2. Consider an uncertain nonlinear system with time-delay as follows:

$$\begin{cases} \dot{x}_1(t) = -(3+\cos^2 x_2(t))x_1(t) + x_2(t) - 0.1\sin^2 x_2(t)x_1(t-\tau) \\ \qquad -(5-2\sin^2 x_2(t))x_2(t-\tau) + c(t)x_2(t)\sin^2 x_2(t) + c(t)x_1(t)\cos^2 x_2(t) \\ \qquad +(1+\sin^2 x_2(t))\omega(t), \\ \dot{x}_2(t) = -(0.4-0.5\cos^2 x_2(t))x_1(t) - x_2(t) + (0.1-0.2\cos^2 x_2(t))x_1(t-\tau) \\ \qquad -0.1x_2(t-\tau) + u(t), \end{cases} \tag{29}$$

where $c(t)$ is an uncertain parameter satisfying $c(t) \in [-0.2, 0.2]$. If we select the membership function as $M_1(x_2(t)) = \sin^2(x_2(t))$ and $M_2(x_2(t))$

$= \cos^2(x_2(t))$, then the nonlinear time-delay system (29) can be represented by the following uncertain time-delay $T-S$ model

Plant Rule 1:

If $x_2(t)$ is M_1, then

$$\begin{cases} \dot{x}(t) = (A_{11} + \Delta A_{11}(t))x(t) + A_{12}x(t-\tau) + B_1 u(t) + B_{\omega 1}\omega(t), \\ \tilde{z}(t) = C_{11}x(t) + C_{12}x(t-\tau) + D_1 u(t), \end{cases}$$

Plant Rule 2:

If $x_2(t)$ is M_2, then

$$\begin{cases} \dot{x}(t) = (A_{21} + \Delta A_{21}(t))x(t) + A_{22}x(t-\tau) + B_2 u(t) + B_{\omega 2}\omega(t), \\ \tilde{z}(t) = C_{21}x(t) + C_{22}x(t-\tau) + D_2 u(t), \end{cases}$$

where

$$A_{11} = \begin{bmatrix} -3 & 1 \\ -0.4 & -1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.1 & -3 \\ 0.1 & -0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{\omega 1} = \begin{bmatrix} 2 \\ 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -4 & 1 \\ 0.1 & -1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & -5 \\ -0.1 & -0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{\omega 2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$C_{11} = C_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C_{12} = C_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$U_1 = U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}.$$

Choose the H_∞ performance level $\gamma = 1$, given the state feedback controller as (7). Then for $\tau = 0.8$, according to Theorem 5, by solving LMIs (26) and (27) , we have

$$X = \begin{bmatrix} 11.6923 & 0.0146 \\ 0.0146 & 0.4336 \end{bmatrix}, \quad M = \begin{bmatrix} 52.0023 & -0.0397 \\ -0.0397 & 1.3720 \end{bmatrix},$$

$$N = \begin{bmatrix} 79.1302 & -2.5054 \\ -2.5054 & 1.5081 \end{bmatrix},$$

and

$$Y_1 = [1.2904 \quad -0.2493], \quad Y_2 = [0.1588 \quad -0.2271].$$

Finally, the responding control gains are given by:

$$K_1 = [0.1111 \quad -0.5788], \quad K_2 = [0.0142 \quad -0.5243],$$

and the maximal delay is $\tau_M = 1.16$.

6. Conclusion

In this paper, a class of uncertain $T - S$ fuzzy-model-based systems with time-delay are considered. Based on Lyapunov functional approach, some new delay-dependent criteria are derived for stabilization and robust H_∞ control of this kind of systems. All the results are given in terms of LMIs. Numerical examples are presented to demonstrate the effectiveness of our methods.

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